OCR

Monday 19 May 2014 – Morning

AS GCE MATHEMATICS

4721/01 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4721/01
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

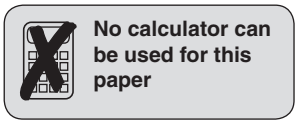
These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



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1 Express $5x^2 + 10x + 2$ in the form $p(x+q)^2 + r$, where p, q and r are integers. **[4]**

2 Express each of the following in the form $k\sqrt{3}$, where k is an integer.

(i) $\dfrac{6}{\sqrt{3}}$ **[1]**

(ii) $10\sqrt{3} - 6\sqrt{27}$ **[2]**

(iii) $3^{\frac{5}{2}}$ **[2]**

3 Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

4 The curve $y = \mathrm{f}(x)$ passes through the point P with coordinates (2, 5).

(i) State the coordinates of the point corresponding to P on the curve $y = \mathrm{f}(x) + 2$. **[1]**

(ii) State the coordinates of the point corresponding to P on the curve $y = \mathrm{f}(2x)$. **[1]**

(iii) Describe the transformation that transforms the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{f}(x+4)$. **[2]**

5 Solve the following inequalities.

(i) $5 < 6x + 3 < 14$ **[3]**

(ii) $x(3x - 13) \geqslant 10$ **[5]**

6 Given that $y = 6x^3 + \dfrac{4}{\sqrt{x}} + 5x$, find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, **[4]**

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **[2]**

7 A is the point (5, 7) and B is the point (–1, –5).

(i) Find the coordinates of the mid-point of the line segment AB. **[2]**

(ii) Find an equation of the line through A that is perpendicular to the line segment AB, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[5]**

8 A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$.

(i) Verify that the curve has a stationary point when $x = 1$. **[5]**

(ii) Determine the nature of this stationary point. **[2]**

(iii) The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the coordinates of Q. **[2]**

9 A circle with centre C has equation $(x-2)^2 + (y+5)^2 = 25$.

(i) Show that no part of the circle lies above the x-axis. **[3]**

(ii) The point P has coordinates $(6, k)$ and lies inside the circle. Find the set of possible values of k. **[5]**

(iii) Prove that the line $2y = x$ does not meet the circle. **[4]**

10 A curve has equation $y = (x+2)^2(2x-3)$.

(i) Sketch the curve, giving the coordinates of all points of intersection with the axes. **[3]**

(ii) Find an equation of the tangent to the curve at the point where $x = -1$. Give your answer in the form $ax + by + c = 0$. **[9]**

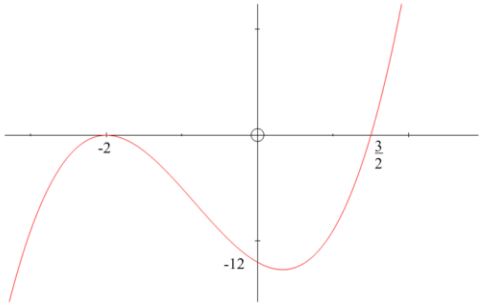
END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$5x^2 + 10x + 2 = 5(x^2 + 2x) + 2$ $= 5[(x + 1)^2 - 1] + 2$ $= 5(x + 1)^2 - 3$	**B1** **B1** **M1** **A1** **[4]**	$p = 5$ $q = 1$ $2 - 5$ "their q"^{2} or $\frac{2}{5}$ – "their q"^{2} **Must be evidence of squaring** $r = -3$	If p, q and r found correctly, then **ISW** slips in format. $5(x + 1)^2 + 3$ **B1 B1 M0 A0** $5(x + 1) - 3$ **B1 B1 M1 A1 (BOD)** $5(x + 1x)^2 - 3$ **B1 B0 M1 A0** $5(x^2 + 1)^2 - 3$ **B1 B0 M1 A0** $5(x - 1)^2 - 3$ **B1 B0 M1 A0** $5x(x + 1)^2 - 3$ **B0 B1M1A0**
2	i)		$2\sqrt{3}$	**B1** **[1]**	**cao**	Do not accept $\frac{6\sqrt{3}}{3}$
	ii)		$10\sqrt{3} - 18\sqrt{3}$ $-8\sqrt{3}$	**B1** **B1** **[2]**	$\sqrt{27} = 3\sqrt{3}$ **soi**, not just $\sqrt{9}\sqrt{3}$	
	iii)		$3^{\frac{5}{2}} = 3^2 \times 3^{\frac{1}{2}}$ $9\sqrt{3}$	**B1** **B1** **[2]**	Separate $\sqrt{3}$ from $3^{\frac{5}{2}}$	Allow **only** $3 \times 3 \times 3^{\frac{1}{2}}$, $3^2 \times \sqrt{3}, 3 \times 3 \times \sqrt{3}$, or $\sqrt{81}\sqrt{3}$, $3\sqrt{9}\sqrt{3}$ for first mark
3			$k = x^2$ $4k^2 + 3k - 1 = 0$ $(4k - 1)(k + 1) = 0$ $k = \frac{1}{4}, k = -1$ $x = \pm\sqrt{\frac{1}{4}}$ $x = \pm\frac{1}{2}$	**M1*** **M1dep*** **A1** **M1** **A1** **[5]**	Substitute for x^2 Attempt to solve resulting quadratic Correct values of k **soi** Attempt to square root Final answers correct, no extras	**No marks** if whole equation square rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no square rooting/squaring at end. If factorising into two brackets: $(4x^2 - 1)(x^2 + 1) = 0$ **M1 A1** $(2x + 1)(2x - 1)(x^2 + 1) = 0$ **M1 A1** **A1** as before Spotted solutions: If **M0 DM0** or **M1 DM0** **SR B1** $x = \frac{1}{2}$ **www** **SC B1** $x = -\frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and

Question			Answer	Marks	Guidance	
						exactly two solutions justified)
4	i)		(2, 7)	B1 [1]		
	ii)		(1, 5)	B1 [1]		
4	iii)		Translation – 4 units parallel to the x axis	B1 B1 [2]	Translation Correct description e.g. correct vector (not as a coordinate), "4 units to the left" **Do not allow second B1 after incorrect type of transformation e.g. stretch/rotation etc. but allow after shift/move etc.**	Do **not** accept shift/move etc. for first **B1** For "parallel to the x axis" allow "horizontally", "in the x direction". **Do not accept** "in/on/ across/up/along/to/towards the x axis". **Do not accept** "factor 4" etc. Allow extra if not incorrect.
5	i)		$5-3< 6x < 14-3$ $2< 6x < 11$ $\frac{1}{3} < x < \frac{11}{6}$	M1 A1 A1 [3]	Attempt to solve two equations/inequalities **each involving all 3 terms** 2, 11 seen from correct inequalities **www** Award full marks if initially working with equations but final answer correct.	Allow " $\frac{1}{3} < x$ **and** $x < \frac{11}{6}$ " " $\frac{1}{3} < x$, $x < \frac{11}{6}$ " but do not allow " $\frac{1}{3} < x$ **or** $x < \frac{11}{6}$ "
	ii)		$3x^2 - 13x - 10 \geq 0$ $(3x+2)(x-5) \geq 0$ $x \leq -\frac{2}{3}, x \geq 5$	M1* M1dep* A1 M1 A1 [5]	Expands and rearranges to collect all terms on one side Correct method to find roots $-\frac{2}{3}, 5$ seen as roots Chooses "outside region" for their roots of their quadratic Do not allow strict inequalities for final mark	**See guidance at end of mark scheme** e.g. $-\frac{2}{3} \geq x \geq 5$ scores **M1A0** Allow "$x \leq -\frac{2}{3}, x \geq 5$", "$x \leq -\frac{2}{3}$ **or** $x \geq 5$" but do not allow "$x \leq -\frac{2}{3}$

Question			Answer	Marks	Guidance	
						and $x \geq 5$" **SC** If question "misread" as $x(3x-13) \geq 0$ Roots found as 0, $\frac{13}{3}$ **B1** $x \leq 0, x \geq \frac{13}{3}$ etc. as above **B1, max 2/5**
6	**i)**		$y = 6x^3 + 4x^{-\frac{1}{2}} + 5x$ $\frac{dy}{dx} = 18x^2 - 2x^{-\frac{3}{2}} + 5$	**B1** **M1** **A1** **A1** **[4]**	$\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ **soi** Attempt to differentiate, any term correct Two correct terms Fully correct, no "+c"	
6	**ii)**		$\frac{d^2y}{dx^2} = 36x + 3x^{-\frac{5}{2}}$	**M1** **A1** **[2]**	Attempt to differentiate their $\frac{dy}{dx}$ **cao www in either part**	Any term still involving x correct – follow through from their expression for the **M** mark only
7	**i)**		$\left(\frac{5+-1}{2}, \frac{7+-5}{2}\right)$ (2, 1)	**M1** **A1** **[2]**	Correct method to find midpoint of line	At least 3 out of 4 terms correctly substituted
	ii)		Gradient of AB $= \frac{7--5}{5--1} = 2$ Perpendicular gradient $= -\frac{1}{2}$ $y - 7 = -\frac{1}{2}(x-5)$ $x + 2y - 19 = 0$	**B1** **B1ft** **M1** **A1ft** **A1** **[5]**	Gradient of AB correctly found as 2 Fully processed $\frac{-1}{\text{their gradient}}$ Equation of straight line through A or B, any non-zero gradient Equation of straight line through A **only**, their perpendicular gradient, in any form Correct equation in given form	i.e. $k(x+2y-19)=0$ for integer k. Must have "=0".

Question			Answer	Marks	Guidance	
8	i)		$\frac{dy}{dx} = 9x^2 - 7 - 2x^{-2}$ When $x = 1$, $\frac{dy}{dx} = 9 - 7 - 2 = 0$ Therefore a stationary point	**M1*** **A1** **A1** **M1dep** **A1** **[5]**	Attempt to differentiate, any term correct Two correct terms Fully correct Substitute $x = 1$ into their derivative Correctly obtain zero **www** and state conclusion **AG**	<u>Alternative for the last two marks:</u> Sets derivative to zero and makes valid attempt to solve resulting quartic **M1dep** Correctly establishes $x = 1$ as solution and draws clear conclusion **A1www**
8	ii)		$\frac{d^2y}{dx^2} = 18x + 4x^{-3}$ When $x = 1$, $\frac{d^2y}{dx^2} > 0$ so minimum	**M1** **A1** **[2]**	Correct method to find nature of stationary point e.g. substituting $x = 1$ into second derivative (at least one term correct from their first derivative in (i)) No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 22.	**Alternate valid methods include:** 1) Evaluating gradient at either side of 1($x > 0$) 2) Evaluating y at 1 and either side of 1 ($x > 0$) If using alternatives, working must be fully correct to obtain the **A** mark
8	iii)		When $x=1$, $y = -2$ $(0, -2)$	**B1** **B1** **[2]**	Finding $y = -2$ at $x = 1$ Correct coordinate **www**	
9	i)		y coordinate of the centre is -5 Radius = 5 Centre is five units below x axis and radius is five, so just touches the x-axis	**B1** **B1** **B1** **[3]**	Correct y value Correct radius Correct explanation based on the above – allow clear diagram **www**	<u>Alt</u> Shows only meets x axis at one point **B1** Correct y value for the centre **B1** Correct explanation **B1 www**
9	ii)		$CP^2 = (6-2)^2 + (k+5)^2$ $CP^2 < 25 \Rightarrow 16 + k^2 + 10k + 25 < 25$ $k^2 + 10k + 16 < 0$ $(k+2)(k+8) < 0$ $-8 < k < -2$	**M1** **A1** **A1** **M1** **A1** **[5]**	Attempt to find CP or CP^2 Correct three term quadratic expression* $k = -2$ and $k = -8$ found Chooses "inside region" for their roots of their quadratic Must be strict inequalities for the A mark * Or $(k+5)^2 < 9$	<u>Alternative</u> Puts $x = 6$ to into equation of circle **M1** Correct three term quadratic equation*, could be in terms of y **A1** $k = -2$ and $k = -8$ found (allow y) **A1** Then as main scheme * Or $(k+5)^2 = 9$ **SC** Trial and improvement **B2** if final answer correct (**B1** if inequalities are not strict)

Question			Answer	Marks	Guidance	
						Can only get 5/5 if fully explained
	iii)		$(2y-2)^2+(y+5)^2=25$ $5y^2+2y+4=0$ $b^2-4ac=4-4\times5\times4$ $=-76$ <0 ,so line and circle do not meet	**M1*** **A1** **M1dep*** **A1** **[4]**	Attempts to eliminate x or y from equation of circle Correct three term quadratic obtained Correct method to establish quadratic has no roots e.g. considers value of b^2-4ac, tries to find roots from quadratic formula Correct clear conclusion **www AG**	If y eliminated: $5x^2+4x+16=0$ $b^2-4ac=16-4\times5\times16$ $=-304$ **No marks for purely graphical attempts**
10	i)			**B1** **B1** **B1** **[3]**	Positive cubic with max and min Correct y intercept – graph must be drawn Double root shown at $x=-2$ and single root at $x=\frac{3}{2}$ with no extras – graph must be drawn	For first mark must clearly be a cubic – must not stop at either axis, do not allow straight line sections/tending to extra turning points etc.
	ii)		x^2+4x+4 or $2x^2+x-6$ $2x^3+5x^2-4x-12$ $\frac{dy}{dx}=6x^2+10x-4$ When $x=-1$, gradient $=-8$ When $x=-1$, $y=-5$ $y+5=-8(x+1)$ $8x+y+13=0$	**B1** **M1** **A1** **M1*** **M1dep*** **A1ft** **B1** **M1** **A1**	Obtain one quadratic factor Multiply their three term quadratic by linear factor to obtain at least 5 term cubic If simplified, must be correct Attempt to differentiate (power of at least one term involving x reduced by one) Substitutes to find gradient at $x=-1$ Correct gradient found **ft** their derivative, differentiation of their expression must be fully correct to earn this mark Correct y value Correct equation of straight line through $(-1$, their $y)$, their gradient from differentiation	**Check for working for this in 10 (i)** <u>Alternative using product rule:</u> Clear attempt at product rule **M1*** Differentiates $(x+2)^2$ correctly **A1** Both expressions fully correct **A2 (1 each)**, then as main scheme y must have been found, do not allow use of gradient of normal instead of tangent i.e. $k(8x+y+13)=0$. Must have "=0".

Question			Answer	Marks	Guidance	
				[9]	Correct answer in correct form	Note If $x = 1$ used instead of $x = -1$, then max possible from last 5 marks is **M1 M1** only

APPENDIX 1

Solving a quadratic

This is particularly important to mark correctly as it features several times on the paper.
Consider the equation: $3x^2 - 13x - 10 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x + 5)(x - 2)$	**M1**	$3x^2$ and -10 obtained from expansion
$(3x - 4)(x - 3)$	**M1**	$3x^2$ and $-13x$ obtained from expansion
$(3x + 5)(x + 2)$	**M0**	only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula
a) If the formula is quoted incorrectly then **M0**.
b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-13 \pm \sqrt{(-13)^2 - 4 \times 3 \times -10}}{2 \times 3}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{13 \pm \sqrt{(-13)^2 - 4 \times 3 \times 10}}{2 \times 3}$$ earns **M1** (10 for c instead of -10 is the only sign slip)

$$\frac{-13 \pm \sqrt{(-13)^2 - 4 \times 3 \times 10}}{2 \times 3}$$ **M0** (2 sign errors: initial sign and c incorrect)

$$\frac{13 \pm \sqrt{(-13)^2 - 4 \times 3 \times -10}}{2 \times -10}$$ **M0** ($2c$ on the denominator instead of $2a$)

Notes – for equations such as $3x^2 - 13x - 10 = 0$, then $b^2 = 13^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving $\pm$; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$3x^2 - 13x - 10 = 0$$
$$3\left(x^2 - \frac{13}{3}x\right) - 10 = 0$$
$$3\left[\left(x - \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10 = 0$$
$$\left(x - \frac{13}{6}\right)^2 = \frac{289}{36}$$
$$x - \frac{13}{6} = \pm\sqrt{\frac{289}{36}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{13}{6}$ seen or implied